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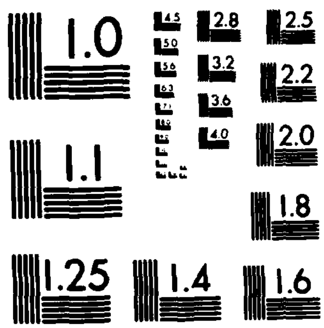

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CONTROLLED APPROXIMATION AND A CHARACTERIZATION OF THE LOCAL APPROXIMATION ORDER

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CONTROLLED APPROXIMATION AND A CHARACTERIZATION OF THE LOCAL APPROXIMATION ORDER

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ABSTRACT

The local approximation order from a scale (S_h) of approximating functions on $\mathbb{R}^m$ is characterized in terms of the linear span (and its Fourier transform) of the finitely many compactly supported functions φ whose integer translates $\varphi(\cdot-j)$, $j \in \mathbb{Z}^m$, span the space $S = S_1$ from which the scale is derived. This provides a correction of similar results stated, and proved in part, by Strang and Fix.

AMS (MOS) Subject Classifications: 41A25, 41A15, 41A63, 65N30

Key Words: Controlled Approximation, Approximation Order, Multivariate, Box Splines, Finite Element Analysis, Fourier Series

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SIGNIFICANCE AND EXPLANATION

In multivariate approximation, we are still groping our way toward an understanding of just what makes a given family of approximants do a good job. As an aid to further understanding, the people in the field consider simple models of suitable approximants and try to establish under what circumstances such model families can approximate every smooth function well. One such model is that of a scale (S_h) of approximating spaces S_h, with each S_h obtained from some fixed space S by scaling, i.e., by a contraction of the underlying independent variable:

$$S_h := \{ s(\cdot/h) : s \in S \}.$$

This idealizes the situation of a finer and finer mesh. A second idealization is that the space S be the same everywhere, i.e., that S be invariant under integer shifts. While spaces actually used in practice are often more complicated, it is hoped that the knowledge arrived at under these simplifying assumptions will nevertheless be relevant, i.e., give a feel for what is to be expected in practical situations.

A very simple model of an effective scale (S_h) of approximants on $\mathbb{R}^m$ is the following: S is spanned by the integer-translates of one compactly supported function φ, and the space S_h is obtained from it by scaling, i.e.,

$$S_h := \{ \Sigma_{j\in \mathbb{Z}^m} a(j)\, \varphi(\cdot/h - j) \} .$$

The approximation order obtainable from such a scale is, by definition, the largest k so that

$$\mathrm{dist}(g, S_h) = O(h^k) \quad \text{for all smooth } g ,$$

with the distance measured in some suitable norm.

Around 1970, several authors concerned with an analysis of the Finite Element method characterized in several ways the functions φ whose associated scale provides approximation order k. These results are gathered in papers by Fix and Strang. The latter go further, though, and consider also the case of a scale for which S is the space of translates of not just one, but of several compactly supported functions, a case of more direct practical interest in several variables where even rather simple approximants have this more complex structure. Strang and Fix intend to characterize controlled approximation order, an order in which the size of the coefficients $a(j)$ of the approximant is to be constrained by the size of g. But they do not provide a complete argument for their characterization; in fact, their characterization is wrong, as Jia has recently shown.

The present report provides several characterizations of a related but simpler concept, that of local approximation order, in which the approximant is constrained to have nonzero coefficients only for $\varphi(\cdot/h - j)$ with support near the support of g. This provides justification for recent work by Dahmen and Micchelli and by Jia concerning the approximation order of spans of box splines.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

CONTROLLED APPROXIMATION AND A CHARACTERIZATION
OF THE LOCAL APPROXIMATION ORDER

C. de Boor and R.-q. Jia

The term "controlled approximation" was introduced in 1970 by Strang, in [St]; see also [FS]. It concerns approximations of the form

$$\sum_{\varphi\in\Phi} \varphi * c_\varphi$$

with Φ a finite collection of functions on $\mathbb{R}^m$ of compact support and $\varphi * c$ the function obtained from φ by convolution with some "sequence" $c: \mathbb{R} \longrightarrow \mathbb{Z}^m$, i.e.,

$$\varphi * c \;:=\; \sum_{j\in\mathbb{Z}^m} \varphi(\cdot - j)c(j) \;.$$

If, more generally, c is some function on $\mathbb{R}^m$, we will still just write $\varphi * c$ instead of the correct but more complicated $\varphi * (c|_{\mathbb{Z}^m})$.

The function u to be approximated lies in the Sobolev space $W_p^k(\mathbb{R}^m)$ with norm

$$\|u\|_{k,p} := \sum_{j\le k} |u|_{j,p}$$

where

$$|u|_{j,p} := \sum_{|\alpha|=j} \|D^\alpha u\|_p$$

and

$$\|u\|_p = |u|_{0,p} := \|u\|_{L_p(\mathbb{R}^m)} \;.$$

We denote by $W_{p,c}^k(\mathbb{R}^m)$ the subspace of $W_p^k(\mathbb{R}^m)$ of compactly supported functions.

The approximations are, more explicitly, of the form

$$\sum_{\varphi\in\Phi} \varphi_h * c_\varphi^h$$

with

$$\varphi_h(x) := \varphi(x/h)/h^{m/p} \;.$$

Concerning the degree of approximation to $u \in W_p^k(\mathbb{R}^m)$ achievable by proper choice of the weights c_φ^h, Strang and Fix [SF; Theorem II] state the following result. In its statement and subsequent analysis, the normalized multivariate monomials appear often enough to deserve an abbreviation of their own. We will use

$$[\,]^\alpha$$

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to stand for the function

$$\mathbb{R}^m \longrightarrow \mathbb{R}: x \longmapsto x^\alpha/\alpha!$$

(and will use standard multi-index notation throughout). In particular, $D^\beta[\,]^\alpha = [\,]^{\alpha-\beta}$, and this holds even when $\beta \not\leq \alpha$ since then $[\,]^{\alpha-\beta} = 0$ by convention. Further, π_j will denote the collection of polynomials on $\mathbb{R}^m$ of total degree $\leq j$. Finally, $\hat{f}$ will denote the Fourier transform of f, i.e., $\hat{f}(\xi) := \int_{\mathbb{R}^m} e^{-i\xi x} f(x)dx$, with ξx the scalar product.

Theorem SF. Let Φ be a finite subset of $W^{k-1}_{2,c}(\mathbb{R}^m)$. Then the following are equivalent:

(i) There exists a sequence $(\psi_\alpha)_{|\alpha|<k}$ in span Φ which satisfies

(ia) $\hat{\psi}_0(0) = 1$, $\hat{\psi}_0 = 0$ on $2\pi\mathbb{Z}^m\backslash 0$;

(ib) $\Sigma_{\beta\leq\alpha}[-iD]^\beta \hat{\psi}_{\alpha-\beta} = 0$ on $2\pi\mathbb{Z}^m$ for $0 < |\alpha| < k$.

(ii) There exists a sequence $(\psi_\alpha)_{|\alpha|<k}$ in span Φ which satisfies

$$[\,]^\alpha = \Sigma_{\beta\leq\alpha}\, \psi_{\alpha-\beta} * [\,]^\beta, \text{ for } |\alpha| < k.$$

(iii) There exist some finitely supported c_φ so that $\psi := \Sigma_{\varphi\in\Phi}\, \varphi * c_\varphi$ satisfies

$$\hat{\psi}(0) \neq 0, \text{ but } D^\alpha\hat{\psi} = 0 \text{ on } 2\pi\mathbb{Z}^m\backslash 0 \text{ for } |\alpha| < k.$$

(iv) For each $u \in W^k_2(\mathbb{R}^m)$, there exist weights c^h_φ so that

(iva) $\|u - \Sigma\, \varphi_h * c^h_\varphi\|_{s,2} \leq \text{const}_s\, h^{k-s}|u|_{k,2}$, $s=0,\ldots,k-1$;

(ivb) $\Sigma_\varphi\, \|c^h_\varphi\|_2^{\,2} \leq \text{const}\, \|u\|_2^{\,2}$.

For the very special case when $m = \#\Phi = 1$, such results can already be found in [Sc]. In [SF], the special case when Φ consists of just one function (but m is arbitrary) is treated first (see [SF; Theorem I]) and completely. However, for the general case, [SF] only give a proof for the implications (i) ==> (iii) ==> (iv). In particular, the validity of the implication (iv) ==> (i) has recently been questioned. This was finally settled by [J] who shows by a counterexample that (iv) does <u>not</u> imply (i) in general.

This raises the question of how to modify (iv) to obtain something equivalent to (i). This is a matter of changing the <u>control</u> over the form of the approximation as expressed by (ivb). In this connection it is very useful to recall that [DM] quote Theorem SF in a modified form. The modification of importance here occurs in condition (iv) which they require to hold locally, as follows:

(iv)' <u>For each</u> $u \in W_p^k(\mathbb{R}^m)$ <u>there exist weights</u> c_φ^h <u>so that, for any closed domain</u> $G \subset \mathbb{R}^m$,

$$\text{(iva)'} \qquad \|u - \Sigma_\varphi \varphi_h * c_\varphi^h\|_p(G) \le \text{const } h^k |u|_{k,p}(B_{rh}(G))$$

$$\text{(ivb)'} \qquad \{\Sigma_{G \cap \text{supp}\varphi(\cdot/h - j) \ne \emptyset} |c_\varphi^h(j)|^p\}^{1/p} \le \text{const } \|u\|_p(B_{rh}(G))$$

<u>holds for some</u> const <u>and</u> r <u>independent of</u> h, G, u.

Here, $B_d(G) := \{x \in \mathbb{R}^m : \text{dist}(x,G) < d\}$.

If (iv)' holds, then, with a reference to [St], [DM] say that Φ provides "controlled L_p-approximation of order k". They do not comment on the fact that (iv)' is a strengthening of (iv), and refer to [SF] for a proof of the implication (iv)' ==> (i).

As it turns out, (iv)' does indeed imply (i). But it is the <u>localness</u> rather than the <u>control</u> that does the job. For this reason, we propose here to abandon the notion of "controlled approximation order" in favor of "local approximation order". We say that Φ provides "local L_p-approximation of order k" in case the following condition holds:

(iv)" <u>For each</u> $u \in W_p^k(\mathbb{R}^m)$ <u>there exist weights</u> c_φ^h <u>so that</u>

$$\text{(iva)"} \qquad \|u - \Sigma_\varphi \varphi_h * c_\varphi^h\|_p \le \text{const } h^k |u|_{k,p} \,;$$

$$\text{(ivb)"} \qquad c_\varphi^h(j) = 0 \ \underline{\text{whenever}} \ \text{dist}(jh, \text{supp } u) > r$$

<u>holds for some</u> const <u>and some</u> r <u>independent of</u> h <u>and</u> u, <u>with</u>

$$\varphi_h(x) := \varphi(x/h) .$$

It is clear that (ivb)' is stronger than both (ivb) and (ivb)", but (ivb) and (ivb)" are not comparable. As we are about to show, (iv)" is the right modification of (iv) to give equivalence with (i). This shows that also (iv)' ==> (i) and so validates the version of Theorem SF in [DM].

Theorem. Let Φ be a finite subset of $W^0_{p,c}(\mathbb{R}^m)$. Then the following statements are equivalent:

(1°) There exists a sequence $(\psi_\alpha)_{|\alpha|<k}$ in span Φ which satisfies

(1°a) $\hat{\psi}_0(0) = 1$, $\hat{\psi}_0 = 0$ on $2\pi\mathbb{Z}^m\backslash 0$;

(1°b) $\Sigma_{\beta\leq\alpha} [-iD]^\beta \hat{\psi}_{\alpha-\beta} = 0$ on $2\pi\mathbb{Z}^m\backslash 0$ for $0 < |\alpha| < k$.

(2°) There exists a sequence $(\psi_\alpha)_{|\alpha|<k}$ in span Φ such that

$$[\,]^\alpha - \Sigma_{\beta\leq\alpha} \psi_{\alpha-\beta} * [\,]^\beta \in \pi_{|\alpha|-1} \quad \text{for} \quad |\alpha| < k .$$

(3°) There exist some finitely supported c_φ so that $\psi := \Sigma_{\varphi\in\Phi} \varphi * c_\varphi$ satisfies

$$[\,]^\alpha - \psi * [\,]^\alpha \in \pi_{|\alpha|-1} \quad \text{for} \quad |\alpha| < k .$$

(4°) For all $p \in [1,\infty]$, Φ provides local L_p-approximation order k .

(5°) For some $p \in [1,\infty]$, Φ provides local L_p-approximation order k .

Remarks. Condition (1°) differs from (i) in that the latter requires, additionally, that $\Sigma_{\beta\leq\alpha}[-iD]^\beta\hat{\psi}_{\alpha-\beta} = 0$ at 0 . Already [DM] prove that it is possible to get away with the weaker condition (1°).

Condition (2°) is, offhand, weaker than (ii), but an inductive argument leads from (2°) to (ii).

Condition (3°) seems more useful to us in applications than the (equivalent) condition (iii). We note that induction gives the seemingly stronger statement

(3°)' There exist some finitely supported c_φ so that $\psi := \Sigma_{\varphi\in\Phi} \varphi * c_\varphi$ satisfies

$$\psi * [\,]^\alpha = [\,]^\alpha \quad \text{for} \quad |\alpha| < k .$$

The asserted equivalence between (4•) and (5•) shows that one might as well drop the qualifier "L_p-" and just speak of the local approximation order provided by ϕ .

Proof of the theorem. While the main point of this note is the implication (5•) ==> (1•), we give a proof of the entire implication cycle (1•) ==> (2•) ==> ... ==> (1•). The arguments for (1•) ==> (2•) ==> (3•) are adaptations of those in [SF].

(1•) ==> (2•). Observe that the Fourier transform of $\psi(x - \cdot)[]^{\beta}$ is $[iD]^{\beta}(e^{-i(\cdot)x}\hat{\psi}(-\cdot))$. Hence, by Poisson's Summation formula, we have

$$\psi*[]^{\beta}(x) = \Sigma_j \, [iD]^{\beta}\big(e^{-i\xi x}\,\hat{\psi}(-\xi)\big)\Big|_{\xi=2\pi j} = \Sigma_j \Sigma_{\gamma\le\beta}[x]^{\gamma} e^{-2\pi i j x}[-iD]^{\beta-\gamma}\hat{\psi}(-2\pi j) \; .$$

It follows that

$$\begin{aligned}\Sigma_{\beta\le\alpha}\psi_{\alpha-\beta}*[]^{\beta} &= \Sigma_j \, \Sigma_{\beta\le\alpha} \, \Sigma_{\gamma\le\beta}[]^{\gamma}e^{-2\pi i j()}[-iD]^{\beta-\gamma}\hat{\psi}_{\alpha-\beta}(-2\pi j)\\ &= \Sigma_j e^{-2\pi i j()}\Sigma_{\gamma\le\alpha} \, []^{\gamma} \, \Sigma_{\beta-\gamma\le\alpha-\gamma}[-iD]^{\beta-\gamma}\hat{\psi}_{(\alpha-\gamma)-(\beta-\gamma)}(-2\pi j)\\ &= []^{\alpha} + \Sigma_{\gamma<\alpha} \, []^{\gamma} \, \Sigma_{\delta\le\alpha-\gamma}[-iD]^{\delta}\hat{\psi}_{\alpha-\gamma-\delta}(0) \; ,\end{aligned}$$

the last equation by (1•).

(2•) ==> (3•). With $\gamma := (k,\dots,k)$, the monomial $[]^{\gamma}$ and its integer shifts span the space $\pi_k(\mathbb{R})\times \dots \times\pi_k(\mathbb{R})$. This implies that there exist finitely supported sequences c_{β} so that

$$[]^{\gamma}*c_{\beta} = []^{\gamma-\beta} \quad \text{for} \quad \beta \le \gamma \; .$$

On applying $D^{\gamma-\alpha}$ to both sides, we find that $[]^{\alpha}*c_{\beta} = []^{\alpha-\beta}$, hence

$$c_{\beta}*[]^{\alpha} = []^{\alpha-\beta} \quad \text{on} \quad \mathbb{Z}^m \quad \text{for} \quad \alpha,\, \beta \le \gamma \; .$$

Now set

$$\psi := \Sigma_{|\beta|<k} \; \psi_{\beta}*c_{\beta} \; .$$

Then

$$\psi*[]^{\alpha} = \Sigma_{|\beta|<k}\psi_{\beta}*c_{\beta}*[]^{\alpha} = \Sigma_{|\beta|<k}\psi_{\beta}*[]^{\alpha-\beta} = \Sigma_{\beta\le\alpha}\psi_{\beta}*[]^{\alpha-\beta} \in []^{\alpha} + \pi_{|\alpha|-1} \; ,$$

the inclusion by (2•).

The proof of (3•) ==> (4•) follows the argument for [BH; Corollary to Theorem 6].

This leaves

(5°) ==> (1°). We approximate a tensor product of univariate B-splines, namely the function

$$u(x) := \prod_{\nu=1}^{m} M_{k+1}(x(\nu))$$

with

$$M_{k+1}(t) := [-\frac{k+1}{2}, -\frac{k-1}{2}, \ldots, \frac{k+1}{2}](\cdot - t)_+^k \ , \ t \in \mathbb{R}$$

(see [Sc]). Since $u \in W_p^k(\mathbb{R}^m)$ for any $p \in [1,\infty]$, we can find weights c_α^h so that (iva)" and (ivb)" hold (whatever the p might be). Set

$$u_h := \sum_\alpha \phi_h * c_\alpha^h$$

and consider the Fourier-Laplace transform

$$\hat{f}(z) := \int e^{-izx} f(x)dx \ , \ z \in \mathbb{C}^m \ ,$$

of the error $f := u - u_h$. Since u has compact support, so does u_h by (ivb)", hence so does f, uniformly in h. This means that supp f lies in some ball B_a of finite radius a independently of h. Consequently

$$|\hat{f}(z)| = |\int_{B_a} e^{-izx} f(x)dx| \le e^{a\|\mathrm{Im} z\|} \mathrm{const}_a \|f\|_p$$

while, by (iva)",

$$\|f\|_p \le \mathrm{const}\ h^k |u|_{k,p} \ .$$

This implies that

$$|\hat{f}(z)| \le \mathrm{const}\ h^k \quad \text{for} \quad \|\mathrm{Im} z\| \le \mathrm{const} \ .$$

We can therefore invoke Cauchy's formula (see, e.g., [R]) to get the estimate

(1) $$\|D^\alpha \hat{f}\|_\infty(\mathbb{R}) \le \mathrm{const}\ h^k \ .$$

The Fourier transform of u is well-known (see [Sc]); it is

$$\hat{u}(z) = \prod_{\nu=1}^{m} C(z(\nu))^{k+1}$$

with $C(t) := (\sin t/2)/(t/2)$ Whittaker's Cardinal function. From this we deduce that

(2) $$\hat{u}(0) = 1$$

and

(3) $$\lim_{h \to 0} (D^\alpha \hat{u})(x/h)/h^k = 0 \quad \text{for} \quad x \in \mathbb{R}^m \backslash 0 \ ,$$

hence, by (1), $\hat{u}_h$ must satisfy corresponding conditions.

We now compute $\hat{u}_h$. Since

$$\int e^{-izx}\varphi(x/h - j)dx = h^m \hat{\varphi}(hz)e^{-ihzj} ,$$

we find that

$$\hat{u}_h(z) = \Sigma_\varphi \hat{\varphi}(hz)v^h_{\varphi,0}(z)$$

with

$$v^h_{\varphi,0}(z) := h^m \Sigma_j c^h_\varphi(j)e^{-ihzj} .$$

Thus, from (1) and (2),

(4) $$\lim_{h\to 0} \Sigma_\varphi \hat{\varphi}(0)\ v^h_{\varphi,0}(0) = 1 .$$

Further

$$[D]^\alpha\left(\hat{\varphi}(hz)v^h_{\varphi,0}(z)\right) = \Sigma_{\beta\le\alpha}\ h^{|\beta|}[D]^\beta\hat{\varphi}(hz)\ (-ih)^{|\alpha-\beta|}v^h_{\varphi,\alpha-\beta}(z)$$

with

$$v^h_{\varphi,\gamma}(z) := h^m \Sigma_j c^h_\varphi(j)\ [j]^\gamma e^{-ihzj}$$

each a $(2\pi/h)$-periodic function. (Note that, for $\gamma = 0$, this agrees with the earlier definition, as it should.) This implies that

$$v^h_{\varphi,\gamma}(2\pi j/h) = v^h_{\varphi,\gamma}(0) ,$$

and therefore (1) and (3) give

$$\lim_{h\to 0} \Sigma_\varphi \Sigma_{\beta\le\alpha}\ h^{|\alpha|}(-i)^{|\alpha-\beta|}[D]^\beta\hat{\varphi}(2\pi j)\ v^h_{\varphi,\alpha-\beta}(0)/h^k = 0 ,$$

hence, for $|\alpha| < k$,

(5) $$\lim_{h\to 0} \Sigma_\varphi \Sigma_{\beta\le\alpha}\ [-iD]^\beta\hat{\varphi}(2\pi j)\ v^h_{\varphi,\alpha-\beta}(0) = 0 \quad \text{for} \quad j \in \mathbb{Z}^m\backslash 0 .$$

From (4) and (5), we deduce (1*) as follows. By (4), we cannot have $\hat{\varphi}(0) = 0$ for every $\varphi \in \Phi$. Without loss of generality, we can therefore assume that, for some $\chi \in \Phi$, $\hat{\chi}(0) = 1$ while $\hat{\varphi}(0) = 0$ for all $\varphi \in \Phi\backslash\chi$. In particular, this implies with (4) that $\lim_{h\to 0}v^h_{\chi,0}(0) = 1$. Now consider the space S of all vectors $w = (w_{\varphi,\gamma})$ for which

$$\lim_{h\to 0} \Sigma_\varphi \Sigma_{|\gamma|<k}\ w_{\varphi,\gamma}v^h_{\varphi,\gamma}(0) = 0 .$$

We claim that $S\perp$ contains a vector w' with $w'_{\chi,0} = 1$. Indeed, if $w'_{\chi,0} = 0$ for all $w' \in S\perp$, then $(S\perp)\perp = S$ would contain the unit vector $(\delta_{\varphi\chi}\delta_{\gamma 0})$, hence $\lim_{h\to 0}v^h_{\chi,0}(0) = 0$ would follow.

With this, define

$$\psi_\gamma := \Sigma_\varphi w'_{\varphi,\gamma}\varphi .$$

Then

$$\hat{\psi}_0(0) = \Sigma_\varphi w'_{\varphi,0}\hat{\varphi}(0) = w'_{\chi,0}\hat{\chi}(0) = 1 .$$

Further, from (5),

$$\Sigma_\varphi \Sigma_{\beta\le\alpha} w'_{\varphi,\alpha-\beta}[-iD]^\beta\hat{\varphi}(2\pi j) = 0 \quad \text{for} \quad |\alpha| < k \quad \text{and} \quad j \in \mathbb{Z}^m\backslash 0$$

and this says that

$$\Sigma_{\beta\le\alpha}[-iD]^\beta\hat{\psi}_{\alpha-\beta} = 0 \quad \text{on} \quad 2\pi\mathbb{Z}^m\backslash 0 \quad \text{for} \quad |\alpha| < k ,$$

as we wanted to show. |||

Remark. In contrast to [SF], we assume Φ only to lie in L_p. For this reason, we do not get (iva), i.e., we do not get simultaneous approximation to derivatives. But this is easily obtained under the assumption that Φ lie in an appropriately smoother space, using the quasi-interpolant constructed in the proof of (3*) ==> (4). In this connection, we note that, using (3*) in the equivalent formulation (3*)', such a quasi-interpolant for u takes the particularly simple form

$$\psi * u .$$

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The local approximation order from a scale (S_h) of approximating [...] on $\mathbb{R}^m$ is characterized in terms of the linear span (and its Fourier tr[...] of the finitely many compactly supported functions φ whose integer tra[...] $\varphi(\cdot-j)$, $j \in \mathbb{Z}^m$, span the space $S = S_1$ from which the scale is deriv[...] provides a correction of similar results stated, and proved in part, by [...] and Fix.

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